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A GAUGE-INVARIANT TREATMENT OF INDUCED EFFECTS IN BETA-DECAY AND AN ANALYSIS OF THE Li^6 RECOIL SPECTRUM

by

Sung Kyu Kim

Department of Physics
Duke University

Date: July 27, 1965

Approved:

Eugene Greuling
Eugene Greuling, Supervisor

E. G. Bilpuch

L. R. Fortney

F. B. Morinigo

A dissertation submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Arts and Sciences of Duke University

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The gauge-invariance of the effective interaction operator appropriate for small momentum transfer is duly respected. When the Coulomb effect terms are neglected, the recoil momentum spectrum is independent of the weak magnetism coupling effect.

The "induced" tensor coupling strength, b, is estimated from a least squares analysis of the Li^6 recoil data. The ratio of correction factors of the C^{12} momentum spectra from B^{12} and N^{12} beta-decay is computed, and the deviation due to "induced" tensor coupling is determined with the above estimate of b.

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A GAUGE-INVARIANT TREATMENT OF INDUCED EFFECTS IN BETA-DECAY AND AN ANALYSIS OF THE Li^6 RECOIL SPECTRUM

Chapter I

INTRODUCTION

The development of our knowledge of beta-decay has taken a somewhat tortuous path. Although beta-radioactivity was observed as early as late nineteenth century, the fundamental experiments giving direct information on the process were not successfully performed until the years following 1945. The experimental activities of the subsequent years, however, witnessed a new vitality incited by the advent of large nuclear reactors, and the corresponding achievements have been monumental. On the theoretical side, the birth of a successful theory of the fundamental process had to be preceded by the development of quantum field theory. First came Pauli's hypothesis of the neutrino in 1930, reclaiming the validity of the "sacred" laws of energy conservation, momentum conservation, and angular momentum conservation for beta-decay. Then, in 1934, Fermi enunciated a fundamental theory of beta-radioactivity, which is still considered correct in its main outlines. The discovery of parity nonconservation in beta-decay by Lee and Yang in 1956 further extended the original Fermi theory. Contributing also to the difficulty in the development of our understanding of the process of beta-decay has

been the required knowledge of nuclear structure and of the nature of the interaction. The interaction turned out to be a new kind of force in physics, and has been called the weak interaction. The study of beta-decay, therefore, has been intimately linked with the understanding of the weak interaction.

Another memorable year in the history of beta-decay is 1958. In that year Feynman and Gell-Mann presented the theory of the universal Fermi interaction. This theory, in part, specifies that the vector coupling constant in beta-decay is not renormalized by strong interactions. In the same year Gell-Mann, drawing an analogy with electromagnetism, suggested that the conserved vector current (CVC) theory could be tested by observing certain second-order corrections to allowed Gamow-Teller beta-decay spectra. The basic ideas underlying the CVC theory are described in Chapter II.

The subject of beta-decay has been one of the most fruitful fields for the application of invariance principles--fruitful, not because the invariance laws applied, but because some of them did not apply. The violation of parity conservation in beta-decay is the best-known example. In Chapter III we review the postulates of charge symmetry and charge conjugation, and consider the possibility that weak interactions may not be invariant under the G-transformation, the product of charge symmetry and charge conjugation.

The requirement of gauge-invariance has often been overlooked--certainly not intentionally--in the development of a theory. The previous treatment of "induced effects" by Huffaker and Greuling has been

no exception. The correction of this oversight could, of course, be achieved by resorting to the conventional method of imposing gauge-invariance. But we have chosen a new, yet necessarily equivalent,course by introducing gauge-invariant matter fields with a definition of the gauge-invariant derivative operator. In the process we encounter some important properties of electromagnetic potentials. These ideas and the consequence of the gauge-invariant beta-interaction operator are discussed in Chapter IV.

Chapter V presents the beta-decay spectra and electron-neutrino angular correlations for Gamow-Teller transitions calculated by using the spherical tensor formalism, and Chapter VI the corresponding recoil momentum spectra.

In Chapter VII the recoil data of Johnson, Pleasonton, and Carlson for He^6 decay are analyzed by the method of least squares. The estimate of the "induced tensor" coupling constant, b, obtained from the above analysis is used in the calculation of the ratio of N^{12} to B^{12} recoil correction factors.

Chapter II

THE CONSERVED VECTOR CURRENT THEORY IN BETA-DECAY

Significant advances[1-3] have been made in recent years in the understanding of some fundamental characteristics of the weak interactions. If they appear to be mere modifications of the original Fermi[4] and Gamow-Teller[5] theories of nuclear beta-decay, they are definitely of the most fundamental kind, for they enable us to think about the nature of weak interactions in a new way--and, perhaps, even in a deeper way. It is for this reason that we devote a chapter to the discussion of the conserved vector current (CVC) theory of Feynman and Gell-Mann[1].

The most general interaction Hamiltonian density for the beta-decay of a bare nucleon can be written as:

$$\mathcal{H} = \sum_{i=1}^{5} \bar{\psi}_p O_i \psi_n \left[\bar{\psi}_e O_i (C_i + C_i' \gamma_5) \psi_\nu + \bar{\psi}_e O_i (D_i + D_i' \gamma_5) \psi_\nu^C \right] + \text{Hermitian conjugate} . \qquad (1)$$

Here, the quantities O_i are operators transforming like either scalars, vectors, tensors, axial-vectors, or pseudoscalars under Lorentz transformations. The C's and D's are constants which determine the strength of the interaction. The remaining quantities are field operators which have the property of creating or destroying particles, as:

ψ_ν destroys a neutrino or creates an anti-neutrino,

$\bar{\psi}_e$ creates an electron or destroys a positron,

ψ_n destroys a neutron or creates an antineutron,

$\bar{\psi}_p$ creates a proton or destroys an antiproton,

ψ_ν^C destroys an antineutrino or creates a neutrino.

In this structure-free theory, there are, in general, twenty coupling constants which could be complex numbers. Great effort has been put into the experimental determination of these constants. The results of these investigations are presented here, in the spirit of theoretical physics, as the basis assumptions of the beta-decay interaction. Then we can make such simplifications in Eq. (1) as are consistent with the basic assumptions. The assumptions and their implications for beta-decay are:

(i) only vector (V) and axial-vector (A) interactions are present,

(ii) the number of leptons (lepton charge) is conserved, which implies that D_i and D_i' must vanish,

(iii) the neutrino is a two-component left-handed particle, which leads to $C_i' = C_i$,

and

(iv) the interaction is invariant under time reversal, which requires that the coupling constants be real.

Under these conditions, the beta-interaction, Eq. (1), takes the form

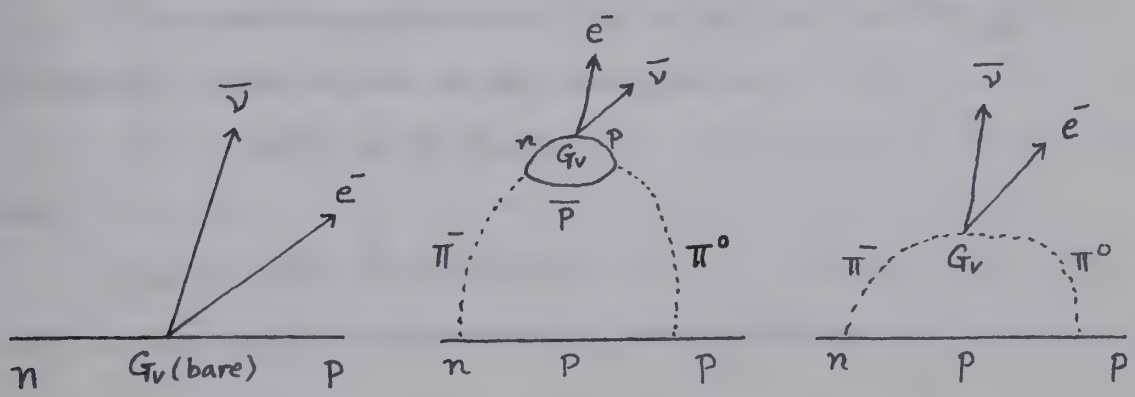
$$\mathcal{H} = (1/\sqrt{2})G[\bar{\psi}_p \gamma_\mu(1 + \gamma_5)\psi_n][\bar{\psi}_e \gamma_\mu(1 + \gamma_5)\psi_\nu] + \text{h.c.}, \quad (2)$$

where G is the Fermi constant. This interaction describes also the beta-decay of the muon. The muon can be considered to be a lepton free of strong interactions. So, we take the interaction Hamiltonian density, Eq. (2), to be the fundamental beta-interaction of a fermion.

Clearly, this structure-free theory can be only approximate for the beta-decay of physical nucleons. If we apply the interaction, Eq. (2), to the physical nucleon which exists surrounded by a pion cloud part of its life-time, it must be assumed that that the nucleon undergoes beta-

decay only when it is in its bare state. Therefore, if a nucleon decays for only a fraction of its life-time, the effective coupling strength of a nucleon must be renormalized by the fraction of time spent as a "dressed" particle. On the other hand, the Fermi interaction strength of a muon needs no renormalization. That is, the effective coupling constant in muon decay should equal the intrinsic one, and it has been found that it does. It would be natural, therefore, to expect a difference in the Fermi interaction strengths for the nucleon and muon decays.

But nature is under no obligation to comply with the expectations of physicists. It was observed--to everyone's amazement--that the effective coupling strengths of the vector interactions in both nuclear beta-decay and muon decay were equal within about 2%. Here was, indeed, a mysterious phenomenon. But mystery is a challenge to physicists. It was Feynman and Gell-Mann[1] who undertook the challenge, and proposed an answer to the question: why is no renormalization required between the effective and the bare interaction strengths of the vector current? They hypothesized that the pions carry with them the same beta-interaction strength as the bare nucleons, when they are virtually emitted from the nucleons. This idea can be graphically expressed in Feynman diagrams such as[6]:



The modifications that arise from the above assumption in the vector current are made in analogy with electrodynamics, which embodies a similar assumption. The assumption is, of course, that the interaction strength of π^+ with the electromagnetic field is the same as that of the proton in a process such as $p \rightleftarrows n + \pi^+$.

The charge current for a nucleon is

$$i_\mu = \bar{\psi}_N \gamma_\mu (1/2)(1 + \tau_z) \psi_N \tag{3}$$

where $\tau_z = +1$ and -1 for proton and neutron, respectively, and ψ_N represents a general nucleon wave function. Clearly, the nucleon current, written in isotopic spin notation, consists of two terms: the first, an isotopic scalar current, which is conserved, thus implying the conservation of the number of nucleons; and the second, the z-component of an isotopic spin vector current, which, as it stands, is not conserved. However, it can be made divergenceless by the inclusion of a pionic term as:

$$J_\mu^z = (1/2) \bar{\psi}_N \gamma_\mu \tau_z \psi_N + \left[\underline{\pi} \times \frac{\partial \underline{\pi}}{\partial x_\mu} \right]_z + \cdots . \tag{4}$$

Thus, we have

$$(\partial / \partial x_\mu) J_\mu^z = 0 . \tag{5}$$

Eq. (5) implies the conservation of charge in strong interactions.

The other two components of the isotopic spin current $\underline{J_\mu}$ are J_μ^+ and J_μ^-. These nucleon currents are given by

$$J_\mu^+ = (1/\sqrt{2}) \bar{\psi}_N \gamma_\mu \tau_+ \psi_N$$

and

$$J_\mu^- = (1/\sqrt{2}) \bar{\psi}_N \gamma_\mu \tau_- \psi_N ,$$

where we have defined the operators τ_+ and τ_- such that

$$\tau_+ \psi_n = \frac{1}{\sqrt{2}} (\tau_x + i\tau_y)\psi_n = \sqrt{2}\,\psi_p ,$$
$$\tau_+ \psi_p = \frac{1}{\sqrt{2}} (\tau_x + i\tau_y)\psi_p = 0 ,$$
$$\tau_- \psi_n = \frac{1}{\sqrt{2}} (\tau_x - i\tau_y)\psi_n = 0 ,$$
$$\tau_- \psi_p = \frac{1}{\sqrt{2}} (\tau_x - i\tau_y)\psi_p = \sqrt{2}\,\psi_n .$$

Of course, the z-component of the isotopic spin current is the familiar electromagnetic isovector current.

We note immediately that J_μ^+ and J_μ^- are just the nucleon currents for the conventional V beta-interaction, which effect β^- and β^+ decay processes, respectively. Feynman and Gell-Mann[1], therefore, suggested that, just as for the electromagnetic isovector current J_μ^z, pionic vector currents be added to J_μ^+ and J_μ^-, so that we have:

$$J_\mu^+ = \frac{1}{\sqrt{2}} \bar{\psi}_N \gamma_\mu \tau_+ \psi_N + \left[\underline{\pi} \times \frac{\partial \underline{\pi}}{\partial x_\mu}\right]_+ + \cdots \qquad (6)$$

$$J_\mu^- = \frac{1}{\sqrt{2}} \bar{\psi}_N \gamma_\mu \tau_- \psi_N + \left[\underline{\pi} \times \frac{\partial \underline{\pi}}{\partial x_\mu}\right]_- + \cdots \qquad (7)$$

thus making each component of the isovector current divergenceless. This is the essence of the conserved vector current (CVC) theory[1,7], and is obviously equivalent to the conservation of total isotopic spin T. (The principle of conservation of T has its origin in the charge-independence of strong interactions.) We have, thus, constructed conserved V beta-currents, and it is their divergencelessness that ensures the conservation of the Fermi coupling constant, i,e., the absence of renormalization by strong interactions in the vector part of the weak interactions.

We can now set up parallel theories for the isotopic vector part of the electromagnetic interaction and the weak vector interaction:

$$\mathcal{H}_V^{e.m.} = e\, J_\mu^z A_\mu$$

$$\mathcal{H}_V^{\beta} = (1/\sqrt{2})\, G\, J_\mu^+ L_\mu .$$

Here L_μ is a lepton current,

$$L_\mu = \bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_\nu .$$

For the situations of our interest we can make certain approximations in the isotopic spin current. For example, for small momentum transfers to the external field, $\underline{\underline{J_\mu}}$ needs not contain the pion field explicitly. In this approximation $\underline{\underline{J_\mu}}$ takes the form[8]

$$\underline{\underline{J_\mu}} \simeq \bar{\psi}_N \gamma_\mu \underline{\underline{\tau}} \psi_N + \left(\frac{i}{e}\right) \bar{\psi}_N \sigma_{\mu\nu} (\kappa_p - \kappa_n) \underline{\underline{\tau}} \psi_N \frac{\partial}{\partial x_\nu} , \quad (8)$$

where κ_p and κ_n are the proton and neutron anomalous magnetic moments, and e is the proton charge.

Gell-Mann[7] called the second term of Eq. (8) "weak magnetism" (WM), because its analogue in the electromagnetic interaction is the anomalous magnetic term. By relating the isovector weak interaction form factors uniquely to the well-known isovector electromagnetic form factors, the WM coupling constant has been estimated to be[7]

$$(\kappa_p - \kappa_n)/2M \simeq 3.7/(2M)$$

in units $\hbar = c = m = 1$. Measurements[9-11] of the beta-decay spectra of N^{12} and B^{12} are in agreement with the above estimate of the WM form factor.

Chapter III

G-CONJUGATION IN WEAK INTERACTIONS

The purpose of this chapter is to sketch the behavior of the beta-interaction under the transformation G, the product of charge symmetry and charge conjugation. Strong interactions are charge symmetric and charge conjugate, and therefore also invariant under the product G[12-14]. However, weak interactions could violate G-invariance. If we allow for the violation of G-invariance in the weak interaction, the beta-currents can be separated into two classes, each with different G-transformation properties[15]. Since in our present state of knowledge there is no evidence either for or against the existence of the second class current, we include both classes in the beta-interaction Hamiltonian. Our presentation consists of a brief review of the charge symmetry and charge conjugation operations in general, followed by a more complete discussion of the operation of G-conjugation in weak interactions.

Charge Symmetry[16]

Charge symmetry is a postulate that asserts that the force between two neutrons is identical to that between two protons if the Coulomb force is neglected. This postulate appears to be well supported by experimental evidence, e.g., the approximate equality of the binding energies and the

Chapter III

[illegible] IN WEAK INTERACTIONS

similarity of the level structures of mirror nuclei. It must be pointed out, however, that the symmetry between neutron and proton is approximate to the extent that it is obviously violated by (i) Coulomb effects and (ii) neutron-proton mass difference. But the postulate of charge symmetry is entirely applicable in nuclear physics, because the Coulomb force is much weaker than the nuclear force and the mass difference is much smaller than the nucleon mass.

Because of the symmetry between them we regard proton and neutron as two distinct states of the same particle, the nucleon. The non-degeneracy which makes them different particles is assumed to disappear if we switch off the Coulomb field. Since the non-degeneracy is a doublet, we take the two states of the doublet to correspond to the two different orientations of an isotopic spin vector of value 1/2. Then the wave function of a nucleon can be expressed by a two-component wave function just as in the case of the ordinary spin:

$$\psi_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \psi_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

In analogy with spin, we introduce three matrices that connect neutron and proton states, and call them the isotopic spin matrices:

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

In this representation ψ_p and ψ_n are eigenstates of τ_3, i. e.,

$$\tau_3 \psi_p = \psi_p, \quad \tau_3 \psi_n = -\psi_n .$$

Charge is evidently an eigenvalue of the operator,

$$q = e(\tau_3 + 1)/2 .$$

The charge symmetry operator T interchanges neutron and proton,

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[illegible]

[illegible] notation ψ_p and ψ_n are eigenvectors of ξ_3, i.e.,

[illegible]

[illegible]

The charge symmetry means [illegible] interchanges neutron and proton.

and must be a unitary matrix with vanishing diagonal elements. We arbitrarily take:

$$T^{(n)} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = i\tau_2^{(n)}.$$

If we define the isotopic spin vector $\hat{I}$ of a system of A nucleons by

$$\hat{I} = \frac{1}{2}\sum_{n=1}^{A} \hat{\tau}^{(n)} \quad ,$$

then T is written as

$$T = i^A \prod_{n=1}^{A} \tau_z^{(n)} = \prod_{n=1}^{A} \exp[\, i\pi\, \tau_2^{(n)}/2] = \exp[\, i\pi\, I_2\,] .$$

From the similarity of the mathematical structure of the spin space and the isotopic space, we can interpret T as a rotation around the "2" axis in charge space by 180 degrees.

The extension of the postulate of charge symmetry is that of charge independence, which states that the force between any two nucleons depends only on their state of spin and relative orbital angular momentum. Charge independence implies the conservation of total isotopic spin. It is clearly a stronger condition than that of the conservation of only the third component, which is equivalent to the conservation of charge. If we again borrow the language of spin space, the charge independence operation can be interpreted as a rotation around any axis in charge space.

Charge Conjugation[17]

Charge conjugation, C, is an operation that transforms particles into antiparticles and vice versa. This operation is not a Lorentz transformation as are the parity transformation and time reversal. It simply changes the nature of the particles involved, while leaving unchanged their linear and angular momenta, and positions.

In our discussion of charge conjugation we start from the Dirac equation for the electron

$$[\gamma_\mu(\partial_\mu - ieA_\mu) + m]\psi = 0 . \tag{9}$$

Though charge conjugation is defined independently of the representation chosen for the γ_μ in the Dirac equation, we take γ_μ to be the four Hermitian matrices that obey the anticommutation relations

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} .$$

The adjoint equation to Eq. (9) is

$$(\partial_\mu + ieA_\mu)\bar{\psi}\gamma_\mu - m\bar{\psi} = 0 , \tag{10a}$$

or

$$[\gamma_\mu^T(\partial_\mu + ieA_\mu) - m]\bar{\psi}^T = 0 , \tag{10b}$$

where the adjoint spinor $\bar{\psi}$ is defined to be

$$\bar{\psi}(x) = \psi^\dagger(x)\gamma_4 = [\gamma_4^T\psi^{\dagger T}(x)]^T ,$$

$\psi^\dagger(x)$ is the Hermitian conjugate of $\psi(x)$,

and T means a transposition.

The charge conjugate wave functions are obtained by adjoining in a Lorentz invariant way solutions of Eq. (9) to those of Eq. (10).

Then the so-called charge conjugate spinor $\psi^C(x)$ and its adjoint $\bar{\psi}^C(x)$ are represented by

$$\psi^C(x) = C\,\bar{\psi}(x)^T, \tag{11a}$$

and

$$\bar{\psi}^C(x) = [\,C^{-1}\psi(x)\,]^T, \tag{11b}$$

where the matrix C satisfies

$$\gamma_\mu^T = \gamma_\mu^* = -C^{-1}\gamma_\mu C, \tag{12a}$$

or

$$\gamma_\mu = -C\,\gamma_\mu^T C^{-1}. \tag{12b}$$

The γ_μ^T satisfy the same anticommutation relations as do the γ_μ, and so C exists, and has the property of being skew-symmetric:

$$C^T = -C \tag{13}$$

and unitary:

$$C^\dagger C = 1. \tag{14}$$

In order to obtain the equations of motion for the matter field, it is necessary to express the Hamiltonian density entirely in terms of $\psi(x)$ and $\bar{\psi}(x)$, or alternatively, $\psi^C(x)$ and $\bar{\psi}^C(x)$. Therefore, we must examine the transformation properties of such bilinear expressions as

$$\bar{\psi}(x)\,O\,\psi(x)$$

where O is an appropriate function of the γ_μ for scalar, vector, tensor, axial-vector, or pseudoscalar.

According to Eqs. (11) and (13), we have

$$\begin{aligned}\bar{\psi}^C\,O\,\psi^C &= (C^{-1}\psi)^T\,O\,C\bar{\psi}^T \\ &= \psi^T(C^{-1})^T\,O\,C\,\bar{\psi}^T \\ &= -\psi^T C^{-1}\,O\,C\,\bar{\psi}^T.\end{aligned}$$

Hence, using Eq. (12) and the anticommutation properties of ψ and $\bar{\psi}$,

we arrive at the following transformation properties:

$$\bar{\psi}^C \, O \, \psi^C = + \bar{\psi} \, O \, \psi \quad \text{in cases of Scalar, Axial-vector, and Pseudoscalar}$$

$$= - \bar{\psi} \, O \, \psi \quad \text{in the cases of Vector and Tensor.}$$

For the sake of completeness, we write down the charge conjugate boson field ϕ^C. For a charged boson field ϕ the C-transformation is given by:

$$\phi^C(x) = C \, \phi(x) = \phi^*(x) ,$$

$$\phi^*(x)^C = C \, \phi^*(x) = \phi(x) .$$

A neutral boson field remains unaffected, i.e.,

$$C \, \phi(x) = \phi(x) .$$

We conclude our sketch of charge conjugation with the remark that, in the interaction representation, the commutation relations and vacuum expectation values are invariant under the C-transformation.

The G-Conjagation Operation in Weak Interactions

The product[12-14] of charge symmetry and charge conjugation is called G-conjugation,

$$G = C T = C e^{i\pi I_2} .$$

Strong interactions are invariant under G-conjugation. This G invariance is of considerable interest to us in that it plays a fundamental role in considering the effects of strong interactions on weak processes[15].

We assume the fundamental beta-interactions to be <u>local</u>. Then the local theory can be represented by[18]:

$$H_\beta = (1/\sqrt{2})\, G \int d^3x\, j_\mu(x)\, L_\mu(x) + h.c., \qquad (15)$$

where $j_\mu(x)$ is a non-local nucleon current and $L_\mu(x)$ is a local electron-neutrino field. The lepton field is assumed to be

$$L_\mu(x) = \bar{\psi}_e(x)\, \gamma_\mu (1 + \gamma_5)\, \psi_\nu(x) . \qquad (16)$$

For the simplest case, i.e., for the bare nucleon, the beta current is given by

$$j_\mu^{\text{"bare"}}(x) = \int d^4x' \left[\bar{\psi}_p(x')\, \gamma_\mu (1 + \gamma_5)\, \delta(s)\, \psi_n(x')\right], \qquad (17)$$

where s is the four-distance between x and x'.

In the case of the "dressed" nucleon, we must include the effects of strong interactions in the beta-current of the nucleon. It follows from the requirement of Lorentz invariance that $j_\mu(x)$ must be of the form[18]:

$$j_\mu(x) = \int d^4x' \left[\bar{\psi}_p(x') \left\{ \gamma_\mu f_V(s) - i\sigma_{\mu\nu} \frac{\partial}{\partial x_\nu} A(s) - \frac{\partial}{\partial x_\mu} C(s) + \gamma_\mu \gamma_5 F_A(s) - i\sigma_{\mu\nu}\gamma_5 \frac{\partial}{\partial x_\nu} B(s) - \gamma_5 \frac{\partial}{\partial x_\mu} D(s) \right\} \psi_n(x')\right], \qquad (18)$$

where the form factors, f_V, f_A, A, B, C, and D, depend only on the four-distance s between x and x'. These form factors are required to be real by time reversal invariance.

In the limit of small momentum transfer, the form factors in the nucleon current may be replaced by products of a constant and a δ-function. Then the current, Eq. (18), reduces to:

$$j_\mu(x) = \int d^4x' \left[\bar{\psi}_p(x')\left\{\gamma_\mu\delta(s) + i\sigma_{\mu\nu}A\frac{\partial\delta(s)}{\partial x_\nu} - C\frac{\partial\delta(s)}{\partial x_\mu} + \lambda\gamma_\mu\gamma_5\delta(s) + i\sigma_{\mu\nu}\gamma_5 B\frac{\partial\delta(s)}{\partial x_\nu} - D\gamma_5\frac{\partial\delta(s)}{\partial x_\mu}\right\}\psi_n(x')\right]. \tag{19}$$

Here, λ is a real constant, i.e., $f_A = \lambda\delta(s)$, which represents the renormalization of the leading term of the axial-vector interaction.

Now, the so-called G-transformation can be applied in order to possibly eliminate some of the form factors in the nucleon current. As for the vector part of the current, the CVC theory requires that it be invariant under the G-parity operation. This implies that the "induced scalar" coupling constant C must vanish. Thus, in the approximation of small momentum transfer, the V current takes the simpler form:

$$j^V_\mu(x) = \int d^4x' \bar{\psi}_p(x')\left[\gamma_\mu\delta(s) + i\sigma_{\mu\nu}A\frac{\partial\delta(s)}{\partial x_\nu}\right]\psi_n(x') . \tag{20}$$

By substituting Eq. (20) for $j_\mu(x)$ in Eq. (15) and integrating by parts $j^V_\mu(x)L_\mu(x)$ over the element d^4x, we obtain the effective Hamiltonian for the V interaction of beta-decay:

$$H^V(x) = (1/\sqrt{2})G\int d^3x\, \bar{\psi}_p(x)\left[\left(1 + i\sigma_{\mu\nu}A\frac{\partial}{\partial x_\nu}\right)L_\mu(x)\right]\psi_n(x) . \tag{21}$$

Here A is the weak magnetism form factor, and its magnitude, in analogy with the CVC theory of electrodynamics, has been estimated to be[7]

$$A = (\kappa_p - \kappa_n)/e = 3.7/(2M) \tag{22}$$

in units $\hbar = c = m = 1$, where $\kappa_p - \kappa_n$ is the difference of proton and neutron anomalous magnetic moments, and M is the nucleon mass.

Whether or not the axial-vector (A) current of the weak interaction is invariant under G-conjugation is still an open question[15]. If it is G-invariant, the "induced tensor" (IT) coupling constant B must vanish. But if B is finite, its magnitude could presumably be determined experimentally[19]. However, its effect in low-energy beta-decay processes would be very small.

The non-conservation of the A current permits the existence of the induced pseudoscalar (IP) coupling. The divergence of the A current, unlike that of the V current, cannot be zero, because that would make the rate of decay of the charged pion vanish[20,21]. The IP coupling term, $D\gamma_5 \frac{\partial L_\mu}{\partial x_\mu}$, may be simplified to $D\gamma_5 \overline{\psi}_e (1 + \gamma_5) \psi_\nu$. That is,

$$D\gamma_5 \frac{\partial}{\partial x_\mu} [\overline{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu] = D\gamma_5 \overline{\psi}_e (1 + \gamma_5) \psi_\nu \equiv D\gamma_5 L_s .$$

This simplification is achieved through the explicit use of the Dirac equations for the electron and neutrino fields. Thus the IP term has the form of the ordinary pseudoscalar interaction with an effective coupling constant $G_P^\beta = G D$ for beta-decay.

For μ-capture, it can be shown in a similar manner that

$$D\gamma_5 \frac{\partial}{\partial x_\mu} [\overline{\psi}_\mu \gamma_\mu (1 + \gamma_5) \psi_\nu] = D\gamma_5 [m_\mu \overline{\psi}_\mu (1 + \gamma_5) \psi_\nu] .$$

The magnitude of the IP coupling constant D has been estimated by Goldberger and Treiman[21] to be

$$G_P^\mu = G D m_\mu \cong 8 G_A \cong - 8 \lambda G$$

where G_A is the axial-vector coupling constant. This magnitude, $G_P^\beta \cong - 0.04 \lambda G$, is practically unobservable in any beta-decay process,

because it follows[22,*] from the Dirac equation for the nucleon and the relation $\underline{\alpha} = -\gamma_5 \underline{\sigma}$ that

$$D\bar{\psi}_N \gamma_5 [\bar{\psi}_e (1+\gamma_5)\psi_\nu] \tau^+ \psi_N = -\frac{D}{2M} [\bar{\psi}_N \gamma_4 \underline{\sigma} \cdot i\underline{\nabla} (\bar{\psi}_e (1+\gamma_5)\psi_\nu) \tau^+ \psi_N + i\bar{\psi}_e (1+\gamma_5)\psi_\nu \frac{\partial}{\partial t} (\bar{\psi}_N \gamma_4 \gamma_5 \tau^+ \psi_N)] . \quad (23)$$

The coefficient (D/2M) is of the order 10^{-5}, and the IP term is clearly negligible in comparison to the small WM term.

If we do not ignore the IT and IP terms, the axial-vector current, in the limit of small momentum transfer, takes the form:

$$j^A_\mu(x) = \int d^4x' \, \bar{\psi}_p(x') \left[(\lambda \gamma_\mu \gamma_5 + i\sigma_{\mu\nu}\gamma_5 B \frac{\partial}{\partial x_\nu} - \gamma_5 D \frac{\partial}{\partial x_\mu}) \delta(s) \right] \psi_n(x') . \quad (24)$$

Thus, we have for the effective Hamiltonian density for the A interaction:

$$\mathcal{H}^A = (1/\sqrt{2}) G \bar{\psi}_p [\lambda \gamma_\mu \gamma_5 + i\sigma_{\mu\nu}\gamma_5 B \frac{\partial}{\partial x_\nu} + \gamma_5 D \frac{\partial}{\partial x_\mu}) L_\mu] \psi_n + \text{H. c.} \quad (25)$$

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* See Appendix A.

Chapter IV

A GAUGE-INVARIANT FORMULATION OF NUCLEAR BETA-DECAY

The effective beta-interaction Hamiltonian density appropriate for small momentum transfer is the sum of Eqs. (21) and (25) :

$$\mathcal{H} = (1/\sqrt{2})\, G\, \bar{\psi}_p\, H\, \psi_n \,. \tag{26}$$

The interaction operator H is given by:

$$H = \left[\gamma_\mu (1 + \lambda\gamma_5) + i\sigma_{\mu\nu} (A + B\gamma_5) \frac{\partial}{\partial x_\nu} + (C + D\gamma_5) \frac{\partial}{\partial x_\mu} \right] L_\mu \tag{27}$$

where L_μ is the lepton current,

$$L_\mu = (\underline{L}, L_4) = \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu \,.$$

Here, λ is the renormalization constant of the leading term of the axial-vector interaction, and A, B, C, and D are, respectively, the weak magnetism (WM), induced tensor (IT), induced scalar (IS), and induced pseudoscalar (IP) coupling constants.

The interaction density $\mathcal{H}$, as written in Eq. (26), is not gauge-invariant. It is well-known, however, that the replacement of the derivative operator $(\partial/\partial x_\mu)$ by $(\frac{\partial}{\partial x_\mu} - ieA_\mu)$ would make the interaction gauge-invariant. This approach is more conventional and certainly valid, but we propose to attack this problem of gauge-invariance in a different manner.

Our aim is to formulate a gauge-independent theory of nuclear beta-decay. Our program consists of introducing the gauge-invariant

matter fields, defining the gauge-invariant differentiation, and writing the interaction in terms of these gauge-invariant field operators.

The gauge-invariant matter fields are [23]:

$$\Psi_p(x,P) = \psi_p(x) \exp\left[-ie\int_{-\infty}^{x} dz_\mu A_\mu(z) \right] \tag{28a}$$

$$\overline{\Psi}_p(x,P) = \overline{\psi}_p(x) \exp\left[ie\int_{-\infty}^{x} dz_\mu A_\mu(z) \right] \tag{28b}$$

$$\Psi_e(x,P) = \psi_e(x) \exp\left[ie\int_{-\infty}^{x} dz_\mu A_\mu(z) \right] \tag{28c}$$

$$\overline{\Psi}_e(x,P) = \overline{\psi}_e(x) \exp\left[-ie\int_{-\infty}^{x} dz_\mu A_\mu(z) \right] \tag{28d}$$

$$\mathcal{L}_\mu(x,P) = L_\mu(x) \exp\left[-ie\int_{-\infty}^{x} dz_\mu A_\mu(z) \right] . \tag{28e}$$

It is obvious that these matter fields are invariant under the gauge transformation:

$$\psi_p' = \psi_p e^{ie\Lambda}$$

$$\psi_e' = \psi_e e^{-ie\Lambda}$$

$$A_\mu' = A_\mu + \frac{\partial \Lambda}{\partial x_\mu}$$

where Λ is some scalar function.

The explicit appearance of P as a variable in the gauge-invariant fields is to emphasize the path-dependent nature of these fields. They are path-dependent by virtue of integration of the 4-vector A_μ along a space-like path P. As a result, these field quantities depend not only on the point x but also on the path itself. By ∞, we mean a spatial infinity which is sufficiently remote in a space-like direction from x so that the electromagnetic field vanishes there. In fact, A_μ may without loss of generality be set equal to zero at that limit. $\Psi(x,P)$ is also single-valued, and since it differs from $\psi(x)$ only in the phase, it can equally well be used in the calculation of quantum mechanical

probabilities.

In order to define differentiation with respect to time, we allow the paths to have infinitesimal time-like portions. Then we can define[23] the gauge-invariant derivatives of $\Psi(x,P)$ by :

$$\frac{\partial}{\partial x_\mu}\Psi(x,P) = \lim_{dx_\mu \to 0} \frac{\Psi(x + dx_\mu, P') - \Psi(x,P)}{dx_\mu} , \quad (29)$$

where the path P' is related to P by $|P' - P| = dx_\mu$.

In terms of the conventional, i.e., path-independent but gauge-dependent, fields, the gauge-invariant derivative of a gauge-invariant matter field $\Psi(x,P)$ is :

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\Psi(x,P) &= \frac{\partial}{\partial x_\mu}\left(\psi(x)\exp\left[-ie\int_{-\infty}^{x}A_\nu(z)\,dz_\nu\right]\right)\\ &= \frac{\partial\psi(x)}{\partial x_\mu}\exp\left[-ie\int_{-\infty}^{x}A_\nu(z)\,dz_\nu\right]\\ &\quad - ieA_\mu\psi(x)\exp\left[-ie\int_{-\infty}^{x}A_\nu(z)\,dz_\nu\right]\\ &= \left[\left(\frac{\partial}{\partial x_\mu} - ieA_\mu\right)\psi(x)\right]\exp\left[-ie\int_{-\infty}^{x}A_\nu(z)\,dz_\nu\right].\end{aligned} \quad (30)$$

Thus, we see that the operation $\left(\frac{\partial}{\partial x_\mu}\right)$ on $\Psi(x,P)$ is equivalent to the operation $\left(\frac{\partial}{\partial x_\mu} - ieA_\mu\right)$ on the conventional field $\psi(x)$ apart from the path-dependent phase factor. That $\left(\frac{\partial}{\partial x_\mu} - ieA_\mu\right)$ is a gauge-invariant operator is, of course, well-known. It is also important to note that $\Psi(x,P)$ obeys the free-particle Dirac equation.

It seems that we have not gained much by introducing the gauge-invariant matter fields in view of the fact that they are now <u>path-dependent</u>. On the other hand, the path-dependence of charged field operators may be a fundamental physical property. This possibility has

been discussed extensively by Aharonov and Bohm[24-26], and has also been demonstrated in an experiment performed by Chambers[27]. (The experiment consisted of separating and recombining a beam of electrons and examining the interference pattern produced.)

The path-dependence of charged fields is explicitly given by the following equations[23]: Suppose we move the path through an infinitesimal area $\sigma_{\mu\nu}$ at a point y, while keeping the end-point x fixed. Then it follows from Stokes' theorem that

$$\begin{aligned} \delta \int_{-\infty}^{x} dz_\mu A_\mu(z) &= [\, \partial_\mu A_\nu(y) - \partial_\nu A_\mu(y) \,]\, \sigma_{\mu\nu} \\ &= F_{\mu\nu}(y)\, \sigma_{\mu\nu} . \end{aligned} \tag{31}$$

Hence the change in the operators $\Psi(x,P)$ and $\overline{\Psi}(x,P)$ caused by such a change in the path is :

$$\delta_y \Psi(x,P) = - ie \Psi(x,P) F_{\mu\nu}(y) \sigma_{\mu\nu} \tag{32a}$$

$$\delta_y \overline{\Psi}(x,P) = ie \overline{\Psi}(x,P) F_{\mu\nu}(y) \sigma_{\mu\nu} . \tag{32b}$$

Clearly, it is the presence of an electromagnetic field $\hat{F}_{\mu\nu}$ that brings about the path-dependence of the field. It is also obvious that if the field variables happen to be independent of the path, the particles in the theory have no interaction with the electromagnetic field. However, even the presence of an electromagnetic field only causes two operators defined at the same point but by different paths to differ merely by a phase factor, and the difference in the phase factor, of course, corresponds to no change in any physical result. The effect of the path-dependence of the operators is essentially quantum-mechanical in nature in that it appears explicitly only in the phenomenon of interference.

The proton and electron fields defined in Eq. (28) can in general have different paths. However, we arbitrarily choose one and the same path for all the field variables that occur in the beta-interaction Hamiltonian. This arbitrary choice of the path is essential, because, otherwise, we must specify a path for each field operator and then perform the path integration for each given path. There is, of course, an infinite number of possible paths, and constraints on these degrees of freedom can only come from the physical conditions. The physical justification for our choice of one and same path for all the field operators involved is simply that the path-dependence of the operators plays an explicit role only when there is an interference phenomenon, and that there is no such interference in beta-decay.

Now we are in a position to write down the beta-interaction Hamiltonian density in the language of gauge-invariant fields. It is :

$$\mathcal{H}(x,P) = (1/\sqrt{2})\, G\, \overline{\Psi_p}(x,P)\, H\, \Psi_n(x) , \tag{33}$$

where H is the same as Eq. (27), except that we have now replaced the ordinary derivative and field operators with the gauge-invariant ones. If we express it in terms of even and odd nucleon operators, retaining the terms that might contribute significantly to Gamow-Teller (GT) β^- spectra, the interaction operator H_{GT} reduces to :

$$\begin{aligned} H_{GT} = {} & i\lambda\beta\underset{\sim}{\sigma}\cdot\underset{\sim}{\mathcal{L}} - i\beta\underset{\sim}{\alpha}\cdot\underset{\sim}{\mathcal{L}} + \lambda\beta\gamma_5\mathcal{L}_4 - iA\underset{\sim}{\sigma}\cdot\underset{\sim}{\nabla}\times\underset{\sim}{\mathcal{L}} \\ & - iB\underset{\sim}{\sigma}\cdot\left(\frac{\partial\underset{\sim}{\mathcal{L}}}{\partial x_4} - \underset{\sim}{\nabla}\mathcal{L}_4\right) + D\gamma_5\left(\underset{\sim}{\nabla}\cdot\underset{\sim}{\mathcal{L}} + \frac{\partial\mathcal{L}_4}{\partial x_4}\right) . \end{aligned} \tag{34}$$

We now eliminate the odd nucleon operators, $\underset{\sim}{\alpha}$ and γ_5 , in H_{GT}

in order to avoid the mixing of the upper and lower components of the nucleon wave function*. We further simplify H_{GT} by use of the non-relativistic approximation $\beta \cong 1$ and the identities

$$\frac{\partial \mathcal{L}_\mu}{\partial x_\mu} = \mathcal{L}_s , \qquad \frac{\partial \mathcal{L}_\mu}{\partial x_4} = (W_o - V)\,\mathcal{L}_\mu , \tag{35}$$

where W_o is the maximum β energy and $V = ieA_4$ is the potential energy of the β particle. Furthermore, we note that the Coulomb field of an electron moving in the field of a nucleus of atomic number Z may be derived from the potentials

$$\underline{A} = 0 , \quad V = - Ze^2/r .$$

Then the resulting GT interaction operator is

$$\begin{aligned} H_{GT} = {} & i\lambda \underline{\sigma}\cdot\{[\,1 - b(W_o - V)]\,\underline{\mathcal{L}} - a\,\underline{\nabla}\times\underline{\mathcal{L}} \\ & + (b + 1/2M)\,\underline{\nabla}\mathcal{L}_4 - d\,\underline{\nabla}\mathcal{L}_s\,\} \\ & - (i/M)\,\underline{\mathcal{L}}\cdot\underline{p} - (\lambda/M)\,\mathcal{L}_4\,\underline{\sigma}\cdot\underline{p} , \end{aligned} \tag{36}$$

where $\underline{p}$ is the nucleon momentum operator and $\mathcal{L}_s$ is a scalar field

$$\mathcal{L}_s \equiv \overline{\Psi}_e\,(1 + \gamma_5)\,\Psi_\nu ,$$

and where we have defined parameters

$$\begin{aligned} a &= (1/\lambda)\,[\,A + (2M)^{-1}\,] , \\ b &= (1/\lambda)\,B \\ d &= (1/\lambda)\,D/2M . \end{aligned} \tag{37}$$

Calculations for beta-decay are customarily done using the conventional gauge-dependent field $\psi(x)$. Our result, Eq. (36), therefore, would look more familiar if we translated it back to the language of the ordinary fields. In terms of conventional field operators, the interaction

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* See Appendix A.

Hamiltonian is :

$$\int \overline{\Psi}_p(x,P)\, H_{GT}\, \Psi_n(x)\, d^3x = \int \overline{\psi}_p(x)\, H'_{GT}\, \psi_n(x)\, d^3x\,, \qquad (38)$$

where we have defined the operator H'_{GT} as

$$H'_{GT} = i\lambda \underset{\sim}{\sigma}\cdot\{[1 - b(W_o - V)]\,\underset{\sim}{L} - a\,\underset{\sim}{\nabla}\times\underset{\sim}{L} + [b + (2M)^{-1}]\,\underset{\sim}{\nabla}\,L_4 - d\,\underset{\sim}{\nabla}\,L_s\} - (i/M)\,\underset{\sim}{L}\cdot\underset{\sim}{p} - (\lambda/M)\,L_4\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,. \qquad (39)$$

We have thus obtained the interaction Hamiltonian operator in terms of the conventional fields.

Chapter V

β-SPECTRA AND ELECTRON-NEUTRINO ANGULAR CORRELATIONS

The Hamiltonian operator, H_{GT}, which has been used for the calculation of the beta-decay spectra[28] and electron-neutrino angular correlations[29] for pure Gamow-Teller transitions, is

$$H_{GT} = i\lambda\underset{\sim}{\sigma}\cdot\{[\,1 - b(W_o - V)\,]\,\underset{\sim}{L} - a\,\underset{\sim}{\nabla}\times\underset{\sim}{L} + (b + 1/2M)\,\underset{\sim}{\nabla}L_4 - d\,\underset{\sim}{\nabla}L_s\} - (i/M)\,\underset{\sim}{L}\cdot\underset{\sim}{p} - (\lambda/M)\,L_4\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p}\ . \tag{40}$$

We recall that in arriving at this form of the GT interaction operator, odd nucleon operators were removed to order m/M (ratio of electron mass to nucleon mass) by means of a Foldy-Wouthuysen transformation[30].

The lepton part of the interaction is expressed in terms of spherical tensors and Coulomb radial wave functions. The four types of spherical tensors needed for our calculations are :

$$S^M_{JL} = \underset{\sim}{T}^M_{JL}\cdot\underset{\sim}{\sigma}\,,$$

$$P^M_{JL} = -(i/M)\,\underset{\sim}{T}^M_{JL}\cdot\underset{\sim}{p}\,,$$

$$W^M_J = -(i/M)\,Y^M_j\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,,$$

where Y^M_J is the usual spherical harmonic and $\underset{\sim}{T}^M_{JL}$ is defined by[31]

$$\underset{\sim}{T}^M_{JL} = \sum_m C(LIJ;M-m,m)\,Y^{M-m}_L\,\underset{\sim}{\xi}_m\,,$$

and $\underset{\sim}{\xi}_m$ is a unit vector in the spherical basis. The reduced nuclear matrix element of an operator such as S^M_{JL} is represented by $\langle S_{JL}\rangle$.

The usual definitions of the lepton wave functions in spherical coordinates are[32]:

$$\psi_e = \psi_\kappa^\mu = \begin{bmatrix} g_\kappa(r)\, \chi_\kappa^\mu(\theta,\phi) \\ i f_\kappa(r)\, \chi_{-\kappa}^\mu(\theta,\phi) \end{bmatrix},$$

$$\psi_\nu = \Phi_\kappa^\mu = \begin{bmatrix} G_\kappa(r)\, \chi_\kappa^\mu(\theta,\phi) \\ i F_\kappa(r)\, \chi_{-\kappa}^\mu(\theta,\phi) \end{bmatrix}.$$

(For a concise discussion of the properties of the angular functions χ_κ^μ and the electron and neutrino radial functions one is referred to Appendix A of the Ph. D. dissertation of J. N. Huffaker[32].)

We define the β spectrum and electron-neutrino angular correlation correction factor $C_o(W,\theta)$ for CT allowed GT transitions as

$$C_o(W,\theta) \propto \frac{N(W,\theta)}{pW(W_o-W)^2\, F_o(W,Z)} . \qquad (41)$$

$N(W,\theta)$ is defined so that, upon integration over the angle θ, $N(W)$ gives the probability of a nucleus beta-decaying in unit time with the electron having energy in an interval dW around W. That is,

$$\text{Probability} = N(W)\, dW .$$

$F_o(W,Z)$ is the Fermi Coulomb function.

The correction factor $C_o(W,\theta)$ to the order 1/M includes the square of the matrix element of the dominant term $i\lambda \underline{\sigma}\cdot\underline{L}$ of H_{GT} and the interference between this term and each smaller term. This amounts to ignoring the terms of order $1/M^2$ or R^4 and to assuming that a, b, and d are of the order 1/M .

We write the GT allowed correction factor $C_o(W,\theta)$ in the form[28]:

$$C_o(W,\theta) = \langle S_{10} \rangle^{-1} \sum_k N_k \langle O_k \rangle \left[C(k) + N'_k C'(k) \cos\theta \cos\Delta \right], \quad (42)$$

where $\cos\Delta = \left[1 + (\alpha Z/p)^2 \right]^{-1/2}$. The radial functions $C'(k)$ may be obtained from the radial functions $C(k)$ by replacing :

$$L_o = \frac{g_{-1}^2 + f_1^2}{2 p^2 F_o} \quad \text{by} \quad L'_o = \frac{2 g_{-1} f_1}{2 p^2 F_o},$$

$$N_o = \frac{f_{-1} g_{-1} - f_1 g_1}{2 r p^2 F_o} \quad \text{by} \quad N'_o = \frac{f_{-1} f_1 - g_{-1} g_1}{2 r p^2 F_o},$$

$$P_o = \frac{g_{-1}^2 - f_1^2}{2 p^2 F_o} \quad \text{by} \quad P'_o = \frac{g_{-1} f_1 - g_{-1} f_1}{2 p^2 F_o},$$

and

$$R_o = \frac{f_{-1} g_{-1} + f_1 g_1}{2 r p^2 F_o} \quad \text{by} \quad R'_o = \frac{f_{-1} f_1 + g_{-1} g_1}{2 r p^2 F_o}.$$

We now go to the $\xi = 0$ approximation for f_κ, g_κ, and various combinations of the electron radial functions. This approximations amounts to ignoring the term αZ, which is much smaller than unity, while keeping potential energy V. Clearly, this limiting process is most useful for light nuclei. If one wishes to obtain the appropriate plane-wave (Z = 0) result from correctly calculated Coulomb spectra, one must first go to the $\xi = 0$ limit and then set V equal to zero.

The approximate spectral functions are[29, 33]:

$$L_o = 1 - (r^2/3) \left[p^2 - 5VW - (V/W) \right],$$

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* Shown in Table I.

$$L_0 = 1 - (r^2/3)\left[\,\cdots - 3\chi_0 - (\chi_0)\,\right],$$

$$P_o = 1/W \ ,$$

$$N_o = (1/2)V - (1/3)(p^2/W) \ ,$$

$$R_o = -(1/2)(V/W) \ ,$$

$$L_o' \cos\Delta = (p/W)\,[\,1 - (1/3)r^2\,(p^2 - 5VW)\,] \ ,$$

$$P_o' \cos\Delta = 0 \ ,$$

$$N_o' \cos\Delta = (p/W)\,[\,V - (W/3)\,] \ ,$$

$$R_o' \cos\Delta = p/3W \ .$$

The GT allowed correction factor $C_o(W,\theta)$ is given in Table I . In the $\xi = 0$ approximation $C_o(W,\theta)$ reduces to :

$$\begin{aligned}
C_o(W,\theta) &= [\,1 - 2(\pm b)(W_o - V)\,]\,[\,1 - \underline{p}\cdot\underline{q}/3WK\,] \\
&+ [\,\pm(4/3)(a + \tfrac{\xi}{2})\,]\,[\,p^2W^{-1} - K - \underline{p}\cdot\underline{q}(W - K)(WK)^{-1}\,] \\
&+ [\,(2/3)(\pm b + 1/2M - \eta\,)]\,[p^2W^{-1} + K + \underline{p}\cdot\underline{q}(W + K)(WK)^{-1}] \\
&- [\,(2/3)\,d\,]\,[\,KW^{-1} + \underline{p}\cdot\underline{q}\,(WK)^{-1}\,] \\
&- [\,r^2/3]\,[\,(p^2 + q^2)(1 - \underline{p}\cdot\underline{q}/3WK) - 2p^2q^2/9WK \\
&\qquad + 2\,\underline{p}\cdot\underline{q}/27 - 4\xi p^2q^2/9WK + 4\xi\,\underline{p}\cdot\underline{q}/9\,] \\
&- V\,[(4/3)(\pm a)]\,[\,1 - \xi/4 - (1 - \xi)\,\underline{p}\cdot\underline{q}/WK\,] \\
&+ V\,[(1/3)(\pm b + 1/2M)]\,[\,1 + 10\,\underline{p}\cdot\underline{q}/WK\,] \\
&- V\,[\,d/3\,]\,[\,17/W\,] \\
&+ V\,[\,r^2/3\,]\,[\,(5p^2 + 6)/W - K/3 - (\underline{p}\cdot\underline{q}/9WK)(15W - 2K) \\
&\qquad - (\xi/3)(\,2K - 4\,\underline{p}\cdot\underline{q}/3W\,)\,] \\
&- V\,[\pm\xi]\,[\,1 - 2\,\underline{p}\cdot\underline{q}/WK\,] \\
&+ V\,[\,\eta\,]\,[\,1 + 2\,\underline{p}\cdot\underline{q}/WK\,]
\end{aligned} \qquad (43)$$

when expressed in terms of the electron and neutrino momenta and energies, $\underline{p}$, $\underline{q}$, W , and K , and of parameters and nuclear matrix element ratios

$$
\begin{aligned}
&\\
&\\
&\\

defined by

$$a = (1/\lambda)[A + (1/2M)],$$

$$b = (1/\lambda)B,$$

$$d = (1/\lambda)(D/2M),$$

and

$$\xi = \sqrt{2}\,\langle S_{12}\rangle\langle S_{10}\rangle^{-1},$$

$$\zeta = \sqrt{2/3}\,\langle r\,P_{11}\rangle\langle S_{10}\rangle^{-1}\lambda^{-1},$$

$$\eta = \sqrt{1/3}\,\langle r\,W_1\rangle\langle S_{10}\rangle^{-1}.$$

Here in the approximate correction factor $C_o(W,\theta)$, upper signs refer to β^- decay, lower signs to β^+ decay.

Table I. Gamow-Teller Allowed Correction Factor $C_o(W,\theta)$ Written in the Form

$$C_o(W,\theta) = \langle S_{10}\rangle^{-1} \sum_k N_k \langle O_k\rangle [C(k) + N'_k C'(k)\cos\theta \cos\Delta].$$

k	N_k	O_k	C(k)
1	$1 \mp 2b(W_o - V)$	S_{10}	L_o
2	$-(1/9)$	$r^2 S_{10}$	$3q^2 L_o + 2qN_o$
3	$-(4/9)$	$2^{1/2} r^2 S_{12}$	qN_o
4	$+(2/3)$	$3^{-1/2} r W_1$	$3N_o - qL_o$
5	$\mp(2/3\lambda)$	$(2/3)^{1/2} r P_{11}$	$3N_o + qL_o$
6	$\pm(4/3)a$	S_{10}	$(2W - W_o - V)L_o - P_o$
7	$(2/3)[\pm b + \frac{1}{2M}]$	S_{10}	$(W_o - V)L_o - P_o$
8	$+(2/3)d$	S_{10}	$-L_o + (2W - W_o - V)P_o$
9	$\pm(2/3)a$	$2^{1/2} S_{12}$	$(W - V)L_o + 3N_o - P_o$
10	$-(2/3)[\pm b + \frac{1}{2M}]$	$2^{1/2} S_{12}$	C(9)
11	$-(2/3)d$	$2^{1/2} S_{12}$	$-L_o + (W - V)P_o + 3R_o$

Upper signs refer to β^- decay, lower signs to β^+ decay.

Table 1. (Cont.)

k	N'_k	C'(k)
1	- (1/3)	L'_o
2	- (1/3)	$3q^2L'_o + 2qN'_o$
3	- (1/3)	qN'_o
4	+ 1	$3N'_o - qL'_o$
5	- 1	$3N'_o + qL'_o$
6	- 1	$(2W - W_o - V)L'_o$
7	+ 1	$(W_o - V)L'_o$
8	+ 1	$- L'_o$
9	- 1	$(W - V)L''_o + 3N'_o$
10	+ 1	C'(9)
11	+ 1	$- L'_o + 3R'_o$

Chapter VI

THE RECOIL MOMENTUM SPECTRUM

In this chapter our primary interest is in the derivation of the distribution function N(Q) of the recoiling ions. The technique employed for the calculation of the recoil momentum spectrum is due to Rose[34]. We also discuss briefly the significance of the vanishing of the term proportional to Gell-Mann's a in the recoil spectrum when the Coulomb effect is neglected.

We begin the derivation of the recoil distribution function N(Q) by first noting that the probability for emission of an electron with momentum $\underset{\sim}{p}$ at an angle θ relative to the neutrino momentum $\underset{\sim}{q}$ is given by

$$N(p,\theta)\, dp\, d(\cos\theta) = \text{const.}\; p^2 q^2 C_o(W,\theta)\, dp\, d(\cos\theta)\,, \tag{44}$$

where $C_o(W,\theta)$ is the spectral and angular correlation correction factor. Eq. (44), by means of the energy relation for electron $W^2 = p^2 + 1$, can be transformed to

$$N(W,\theta)\, dW\, d(\cos\theta) = \text{const.}\; pWq^2 C_o(W,\theta)\, dW\, d(\cos\theta)\,. \tag{45}$$

If we define $\underset{\sim}{Q} = \underset{\sim}{p} + \underset{\sim}{q}$ to be the negative recoil momentum, then $\cos\theta$ can be expressed in terms of Q and W by

$$2pq\cos\theta = Q^2 - p^2 - q^2\,. \tag{46}$$

Now, while holding p and q constant, we let Q be a function of the varying angle θ between the electron and neutrino. Then, Eq. (45) can be written as

$$N(Q,W)\, dQ\, dW = \text{const.}\; WqC_o(W,\theta)\, dW\, QdQ\,, \tag{47}$$

where $\int_{W_-}^{W_+} N(Q,W)\,dW$ is the distribution function of the recoil ions.

Integration over the electron energy W is done with Q fixed. The limits of W-integration are determined in the following manner: From Eq. (46) it follows that

$$0 \leq |p - q| \leq Q \leq p + q \,. \tag{48}$$

In the situation, $Q = p + q = (W^2 - 1)^{1/2} + (W_o - W)$, we see that $Q_{max} = (W_o^2 - 1)^{\frac{1}{2}}$ is obtained when $q = 0$ and $Q_{min} = (W_o - 1)$ when $p = 0$.

Thus we have

$$W_o - 1 \leq Q \leq W_o^2 - 1 \,, \text{ for } Q = p + q \,. \tag{i}$$

For the case $Q = |p - q|$, we have the two possibilities :

a) for $q \geq p$,

$$Q = q - p = W_o - W - (W^2 - 1)^{1/2} \tag{iia}$$

b) for $p \geq q$,

$$Q = p - q = (W^2 - 1)^{1/2} - (W_o - W) \,. \tag{iib}$$

The limits on the W-integration are defined by the three curves--(i), (iia), and (iib)--in the Q-W plane. These curves enclose a region defined by

$$W_- \leq W \leq W_+$$

where the solutions of Eqs. (i) and (iia), and (iib) yield W_- and W_+ , respectively :

$$W_{\pm} = (1/2)\left[W_o \pm Q + (W_o \pm Q)^{-1}\right] \,. \tag{49}$$

In order to obtain the recoil distribution function N(Q), one only needs to integrate Eq. (47) with respect to W between the limits W_- and W_+ .

Given a correction factor $C_o(W,\theta)$, the calculation of the recoil momentum spectrum is now straightforward, though very tedious in some cases. Using the correction factor $C_o(W,\theta)$ for allowed GT transitions, as given by Eq. (43(, we obtain the following result for the recoil spectrum:

$$
\begin{aligned}
N(Q) \;=\; & \left[1 - 2(\pm b)(W_o - V)\right] F_1(Q) \\
& + \left[\pm a\right]\left[\, F_a(Q) \;-\; V\, G_a(Q) \,\right] \\
& + \left[\pm b + \frac{1}{2M}\right]\left[\, F_b(Q) \;-\; V\, G_b(Q) \,\right] \\
& + \left[\, d \,\right]\left[\, F_d(Q) \;-\; V\, G_d(Q) \,\right] \\
& - \left[R^2/3\right]\left[\, F_r(Q) \;-\; V\, G_r(Q) \,\right] \\
& - \left[\, \eta \,\right]\left[\, F_b(Q) \;-\; V\, G_\eta(Q) \,\right] \\
& + \left[\pm \zeta\right]\left[\, (1/2)\, F_a(Q) \;-\; V\, G_\zeta(Q) \,\right] . \qquad (50)
\end{aligned}
$$

Here, the potential energy of the electron V is defined so that $V = -\alpha Z/R$, and R is the nuclear radius given by $R = 0.426\alpha A^{-1/3}$. In terms of the quantity $X = W_o^2 - Q^2$, the various functions of Q occurring in Eq. (50) are :

$$F_1(Q) = (1/18)X^{-3}Q^2(X-1)^2[X(4X-1) + 2(X+2)W_o^2]$$

$$F_a(Q) = 0$$

$$F_b(Q) = (2/9)X^{-3}Q^2(X-1)^2[(X+2)W_oQ^2]$$

$$F_d(Q) = (1/3)X^{-2}Q^4(X-1)^2$$

$$G_a(Q) = (1/12)X^{-3}Q^2(X-1)^2[8X^2 - \xi\{X(7X+1-2W_o^2) - 4W_o^2\}\,]$$

$$G_b(Q) = (1/36)X^{-3}Q^2(X-1)^2[4X(2Q^2-1) + 16W_o^2 + \xi\{19XQ^2 + X(9W_o^2-5)+20W_o^2\}]$$

$$G_d(Q) = (1/6)(2-5\xi)X^{-2}Q^2(X-1)^2W_o$$

$$
\begin{aligned}
F_r(Q) = {} & Q^2\, F_1(Q) \;+\; [(13-3\xi)/1620]X^{-5}Q^2(X-1)\{-4W_o^2Q^2[(X^3+X)(7X-8)-10X+24] \\
& + 12Q^2X(X+1)(X^3+1) + 12W_o^2X^2(5X^2-7X+5) \\
& + X^2[10X(X+1)(X-1)^2 + 12X^2 - 15(X^2+1)^2]\}
\end{aligned}
$$

$$+ (1/9)X^{-2}Q^2(X-1)^2[X(2X-1) + W_o^2Q^2]$$

$$+ (1-\xi)(1/540)X^{-5}Q^2(X-1)\{W_o^2[-60X^2(X^2-3X+1)+8Q^2(X^4+X^3+X^2-9X+6)]$$

$$- [15X^2(3X^4-6X^2-4X-1) - 4Q^2X(3X^4-2X^3-2X^2-2X+3)]\}$$

$$G_r(Q) = - (\xi/54)X^{-3}Q^2(X-1)^2[12X^2-W_o(X+1)(5X-3)+2W_o(2W_o^2+1)Q^2]$$

$$+ (\xi/108)X^{-4}Q^2(X-1)W_o[3X(X^2-1)(X+3) + 12X(X-1)W_o^2(Q^2-3)$$

$$- 6(X+1)(3X^2-4X+3)Q^2 + 4(X^3-3)Q^2]$$

$$+ (2\xi/9)X^{-3}Q^2(X^2-1)W_o^3 + X^{-2}Q^2(X-1)W_o(3X+2)$$

$$+ (1/72)X^{-4}Q^2(X-1)W_o[X(X^2-1)(X+3) + 4X(X-1)W_o^2(Q^2-3)$$

$$- 2(X+1)(3X^2-4X+3)Q^2 - 10W_o(X^3+X^2+X-3)Q^2$$

$$+ 15X(X-1)(X^2-1) + 8W_o^3X(X+1)]$$

$$+ (1/108)X^{-3}Q^2(X-1)^2[-12X^2 + W_o(X+1)(5X-3) - 2W_o(2W_o^2+1)Q^2]$$

$$+ (1/54)X^{-4}Q^4(X-1)W_o(X^3-3)$$

$$G_\eta(Q) = - (1/4)X^{-3}Q^2(X-1)^2[-X(3X+1) + 4W_o^2(X+1)]$$

$$G_\zeta(Q) = (1/12)X^{-3}Q^2(X-1)^2[3X(3X-1) - 3Q^2(X+1) - Q^2] .$$

Clearly, N(Q) vanishes at the end points of the recoil momentum, that is, at Q = 0 and X = 1, which corresponds to $Q_{max} = W_o^2 - 1$. This result is in accord with the experimental requirement that a distribution function of this kind assume the value zero at the extreme points of the variable.

We now discuss the significance of the vanishing of the function $F_a(Q)$. This, of course, implies that the term proportional to Gell-Mann"s a does not affect the recoil spectrum in the plane-wave (Z=0) approximation. A similar result was also obtained[35] in the recoil spectrum for first-forbidden unique transitions.

This interesting and important result is in accord with a theorem stated and proved by Weinberg[15]. The theorem states:

> If we perform any experiment with arbitrarily large momentum transfer, which does not distinguish between the lepton and neutrino and if we may neglect the mass and charge of the lepton, then there can be no interference between the coupling types V and A.

Gell-Mann's weak magnetism term is just the interference coupling between V and A. In the plane-wave approximation the charge of the lepton is neglected. Its mass does not affect the momentum of the recoiling nucleus. Thus, in the recoil experiment, we meet the conditions of Weinber's theorem, as stated above.

When the Coulomb effect is included, however, the Gell-Mann term does not vanish. It does make a contribution to the recoil spectrum, though of a very small amount proportional to the lepton potential energy V. In He^6 beta-decay, it is less than 0.5% of the dominant term.

Chapter VII

ANALYSES OF RECOIL SPECTRA

Recoil experiments have played a prominent role in the development of our understanding of beta radioactivity. It has already been pointed out that the theory of nuclear beta-decay allows for five different covariant interactions, of which interactions S and V can be effective in allowed Fermi transitions, and interactions T and A in allowed Gamow-Teller transitions. The pseudoscalar (P) interaction plays no part in allowed transitions. Clearly, the burden of deciding which of these interactions actually account for the observed natural phenomena must rest with experimentalists.

The most direct way of determining the nature of the beta interaction has been the measurement of the electron-neutrino angular correlation. In an allowed transition where beta-particles are emitted with velocity v, the angular correlation has the form

$$1 + \alpha(v/c)\cos\theta$$

where α is -1, +1, +1/3, and -1/3 for pure S, V, T, and A, respectively. It is, then, obvious that the angle θ between the direction of emission of the neutrino and the beta-ray is very sensitive to the fundamental interaction assumed between the transforming nucleon and the lepton field.

Since the direct observation of the emitted neutrinos is impossible, the direction of the neutrino relative to the beta-ray is inferred from observations of the recoil nucleus. The accurate determination of the relative strengths of the interactions, therefore, depends critically on the precision of recoil spectrometry.

An additional significance has been assigned to angular correlation measurements since the discovery[36,37] of non-conservation of parity in weak interactions. The non-conservation of parity implies that the emitted leptons may be polarized. The two-component neutrino theory[38-40] and the V±A theories[1-3] predict that the polarisations of the $\beta^{\pm}$ particles are exactly $\pm$ v/c and that those of their accompanying massless neutrinos are exactly $\mp$ 100 %. Various experiments[41] have been performed to determine the electron polarization with ever increasing precision. Recent measurements by Brosi et al[42] give a polarization of (-0.990 ± 0.009)v/c for the β^{-} particles from P^{32}. The experimental confirmation of the predicted electron polarization enables one to relate the electron-neutrino angular correlation coefficients to the electron polarization by noting that the coefficients $(-\alpha)$v/c for Fermi and (3α)v/c for GT transitions are the average relative helicities of the electrons.

Analysis of the Li^6 Recoil Spectrum

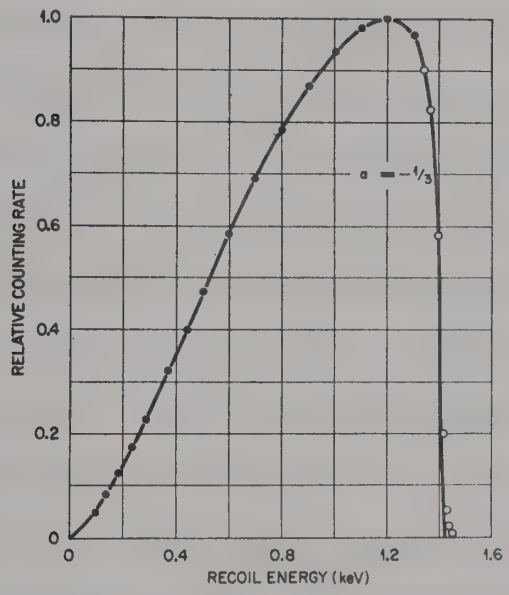
Recoil experiments are extremely difficult to perform due to the low energy--only a few hundred electron volts--of the recoil ions. As a result, one is almost of necessity confined to measurements with rare gases, because in such elements solid state or molecular effects on the recoil ions can be neglected. Recent efforts in the increased precision of recoil spectrometry[19,43,44] have been fruitful, and precision recoil measurements have been successful.

He^6 is a rare gas that undergoes a pure Gamow-Teller transition to a single final state, with a decay energy of 3509 ± 4 kev[45] and a half-life of 0.797 ± 0.003 sec.[46] Johnson *et al*[19] have done a precision measurement of the recoil energy spectrum from the decay of He^6. The measurement was facilitated by making use of the Oak Ridge Research Reactor for the prolific production of He^6, and of an apparatus designed by Snell and Pleasonton[44] for the reduction of the background to a small fraction of the signal. The other well-known difficulties have been recognized and taken into account in the assignment of errors. The analysis of the Li^+ recoil spectrum was carried out by the experimentalists themselves, and has yielded results consistent with the presently accepted V-A theory.

The excellent agreement of the recoil data with the theory is shown in Figure 1. The curve is the plot of the dominant term of the theoretical recoil spectrum, which was calculated using the value -1/3

Figure 1

RECOIL ENERGY SPECTRUM



for the electron-neutrino angular correlation coefficient in accordance with the pure axial-vector theory. The solid dots indicate data points obtained from the observation of the spectrum of singly charged Li^6 recoil ions as a function of the average recoil energy of the analyzed ions. Data shown by open circles near the end point were obtained for the purpose of the end-point analysis. Uncertainties from counting statistics are less than the sizes of the points. Clearly, one cannot detect any significant deviations of the recoil data from the theoretical spectrum calculated from the pure axial-vector theory. If second-order "induced" couplings do exist besides the "weak magnetism" coupling, which has no effect on the recoil spectrum, their effects can only be very small. However, the precision of the measurement is such that information about the higher-order couplings could be elicited from the data.

First, we need a complete recoil momentum spectrum, which, in addition to the dominant term, includes all second-order contributions. Such a spectrum has been calculated, and is given by Eq. (50). The higher-order terms included are the "induced"couplings, the "finite de Broglie wavelength" effect, and the "second-forbidden" nuclear matrix element terms with coefficients η and b. The Coulomb effect enters in through the terms with coefficient V, the potential energy of the electron. If we define a correction factor C(Q) for the recoil momentum spectrum by

$$C(Q) = N(Q)/F_1(Q)$$

where N(Q) is the total spectrum given by Eq. (50) and $F_1(Q)$ the dominant term of N(Q), then the second-order terms effect a deviation of C(Q) from unity.

The recoil data obtained by Johnson et al[19] are actually an energy spectrum. However, it is an easy matter to convert a momentum spectrum to an energy spectrum, since

$$N(Q)\,dQ = N(E)\,dE = \text{const.}\; N(E)\,Q\,dQ.$$

Here $N(E)$ represents the relative abundances of the Li^6 ions as a function of their recoil energy E. The actual measured values are proportional to $E \cdot N(E)$ by virtue of the fact that the resolution width of the spectrometer is a fixed fraction of the energy of the ions being analyzed. If we denote the spectrum $E \cdot N(E)$ by $U(E)$, then

$$U(E) = E \cdot N(E) = \text{const.}\; Q \cdot N(Q) .$$

The actual measurement consisted of taking twelve sets of data with different modes of spectrometer operation. From them a composite spectrum was formed with uncertainties based on the fluctuations among the twelve sets. The composite spectrum of 16 data points $U_k \pm \sigma_k$ is presented in Table II . It is this experimental composite spectrum that we analyze in order to estimate the IT coupling strength B.

We first consider the plane-wave analysis of the recoil data. In this approximation we ignore the Coulomb effect (V), the "finite de Broglie wavelength" effect (R^2), and the "second-forbidden" nuclear matrix element terms (η and ζ), keeping only the "induced" couplings. Such an analysis is qualitatively useful, because in this approximation the recoil spectrum is most sensitive to the IT coupling. Gell-Mann's WM term does not contribute at all, and the IP coupling is negligible.

The results of the plane-wave analysis are presented in Fig. 2. Curve (1) corresponds to the theoretical correction factor with B=0 .

Table II. The Composite Spectrum of Data Points $U_k \pm \sigma_k$ at the 16 recoil Energies E_k (W_o=7.865 and E_k=23.303Q^2, momentum in mc units)

E_k(eV)	U_k	σ_k
99.4	854.1	5.7
139.6	1381.1	8.4
182.4	2037.8	9.0
230.7	2822.8	3.3
285.0	3750.3	15.0
366.2	5232.5	9.8
433.5	6472.9	9.8
496.6	7644	12.5
596.2	9404	8.8
696.0	11116	4.9
795.5	12618	16.0
895.0	13886	13.8
994.5	14899	15.2
1093.2	15613	17.7
1187.8	15923	13.8
1293.0	15381	20.0

Adapted from Reference 19.

The slight 0.3 % rise of the curve is due to the (1/2M) term, which is given by

$$C(Q) = 1 + \left(\frac{1}{2M}\right) \frac{4W_o(X + 2)\, Q^2}{X(4X - 1) + 2(X + 2)\, W_o^2} .$$

The open circles with error flags represent the experimental correction factor $C^{exp}(E)$, which is defined to be

$$C^{exp}(E) = U(E)/D\, Q \cdot F_1(Q)$$

where D is a normalization constant chosen so that $C^{exp}(0.5 \text{ kev}) = 1$. Curve (2) is a least squares fit to the data, from which we obtain $B = -1.1\lambda/M$.

Curve (3) is the theoretical correction factor computed using for B a value that was estimated by Huffaker and Greuling[28] from a theoretical analysis of the data on μ capture in C^{12} and the ratio of B^{12}-N^{12} ft values. It is quite evident that this estimate, $B = -3.3\lambda/M$, is inconsistent with the Li^6 recoil data.

Since the plane-wave estimate of B from a least squares analysis of the Li^6 recoil data is of the same order of magnitude as the ignored terms, it becomes necessary to include all possible higher-order terms for a more conclusive answer. Therefore, we incorporate now the Coulomb and the "finite de Broglie wavelength" effects, and several "second-forbidden" correction terms in the recoil spectrum--all in a gauge-invariant manner.

The following parameters now appear in the recoil spectrum in addition to the previously-treated "induced" coupling constants: the nuclear radius R and nuclear matrix element ratios, ξ, η, and ζ. The nuclear radius is computed from the relation, $R = 0.426\alpha A^{1/3}$,

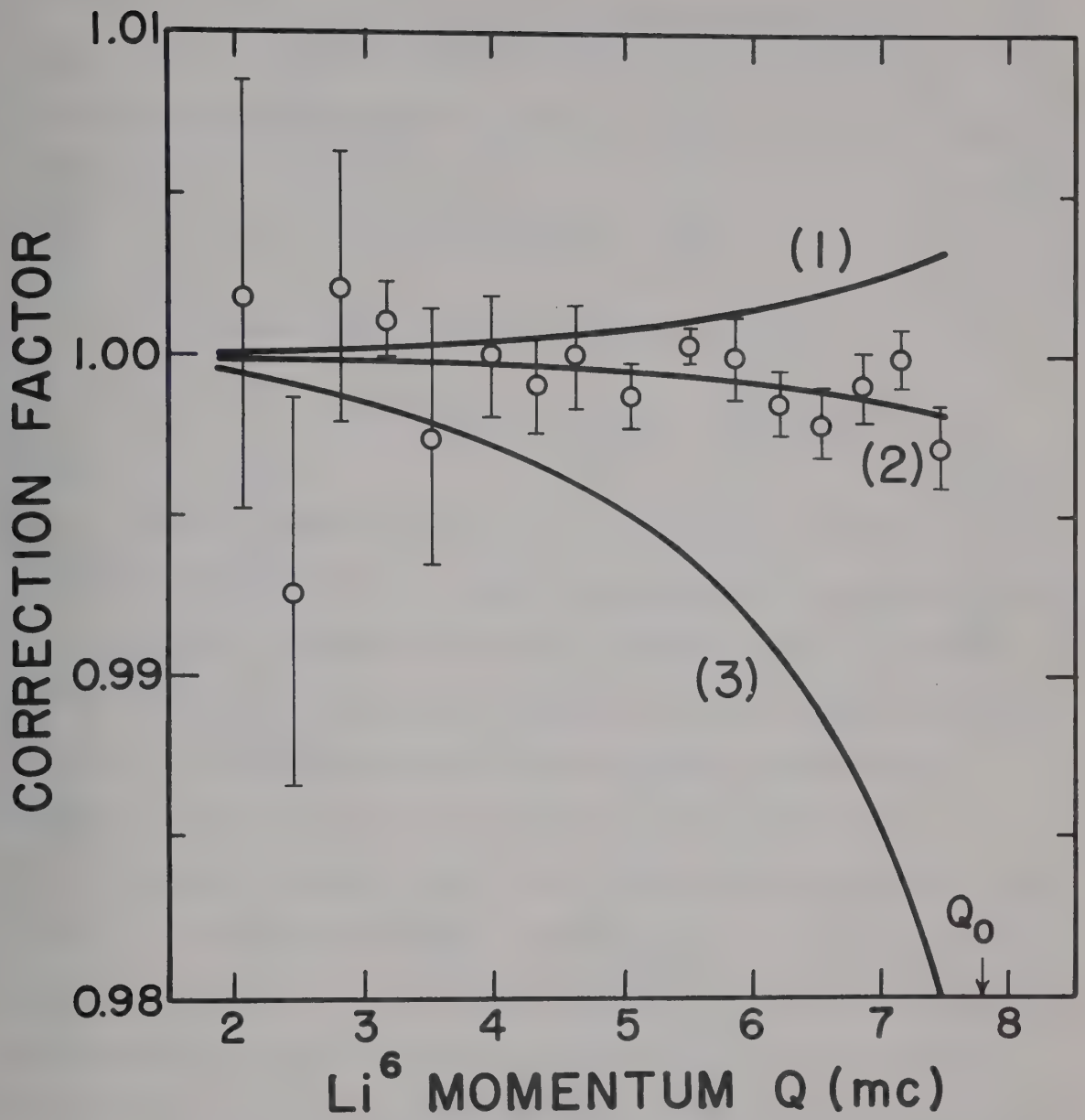


Figure 2. The Li^{6+} recoil ion correction factor from He^6 decay. The solid curves are for the following values of the IT coupling constant: (1) B = 0; (2) B = $-1.1\lambda/M$; (3) B = $-3.3\lambda/M$. The data are normalized to fit the least squares analysis curve (2).

which gives for He^6 a radius of 5.647 (10^{-3}). Using this value of R, the potential energy V of the electron in the Coulomb field of the Li^6 nucleus is $V = -\alpha Z/R = -3.583$. The nuclear matrix element ratios for He^6 have been evaluated[29] from the j-j coupling model. The results are:

$$\xi = 2^{1/2}\langle S_{12}\rangle / \langle S_{10}\rangle = 2/5 ,$$

$$\zeta = (2/3)^{1/2}\langle rP_{11}\rangle / \lambda\langle S_{10}\rangle = 1/\lambda M ,$$

$$\eta = (1/3)^{1/2}\langle rW_1\rangle / \langle S_{10}\rangle = (9/160)|V|R^2\Lambda$$

where $\Lambda = 2 \pm 1$.

Thus we have the values of all the parameters except that of B, the estimate of which is the reason for our analysis. The method employed for the analysis is again that of least squares. When the deviations of the data from the theory are subjected to a least squares analysis, with all 16 experimental points equally weighted, the result is a slightly positive value for B, namely, $B = (0.5 \pm 1.0)\lambda/M$.

Figure 3 shows the results of this all-inclusive analysis of the data. It is immediately evident that the curve for $B = 0$ is a very good fit to the data. If the second data point is ignored, the RMS deviation of the 15 data points from the theoretical curve is only 0.17 %. The upper and lower curves represent the upper and lower limits of the above estimate of B.

It seems that the analysis of the Li^6 recoil data supports the G-invariance of the nucleon-lepton weak interaction. However, more evidence should be required before writing off the existence of the IT coupling altogether.

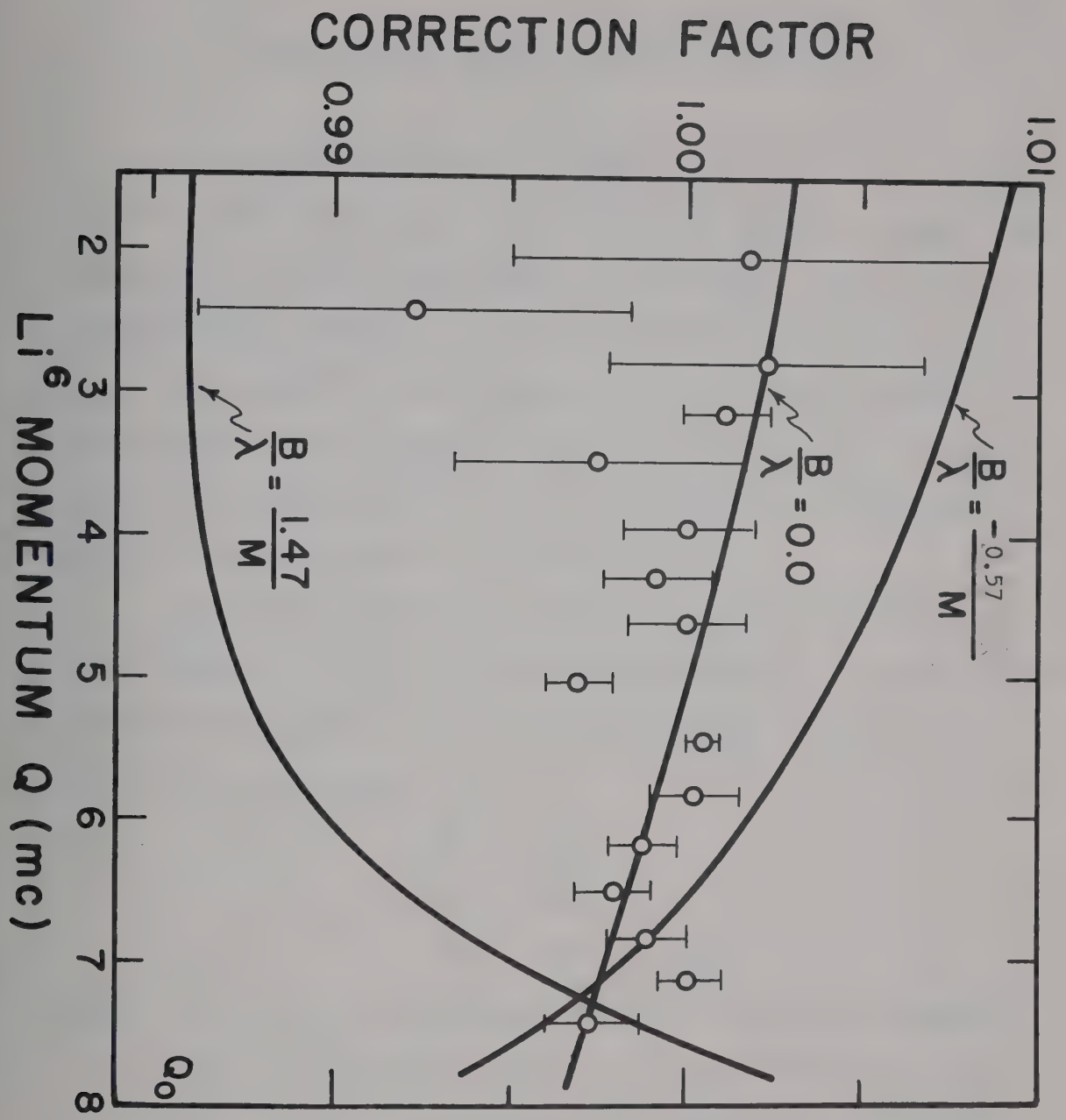


Figure 3

The Li^6 recoil ion correction factor including all second-order terms

The C^{12} Recoil Spectra from N^{12} and B^{12} β-Decay

The suggestion that the β spectra of B^{12} and N^{12} be measured accurately and compared as a test of the conserved vector current (CVC) theory was first made by Gell-Mann[7]. The subsequent experimental measurements[9-11] have been in general agreement with the CVC theory.

The effect of the IT coupling $\underline{b}$ on the recoil spectrum is more pronounced for B^{12} and N^{12} than for He^6 decay. This is due to much higher decay energies of B^{12} and N^{12}-- $27.164(mc^2)$ and $33.152(mc^2)$, respectively--as compared to $7.865(mc^2)$ of He^6.

Parameters that appear in the recoil spectrum are known. The estimates of $\underline{a}$ and $\underline{d}$ have already been cited. The matrix element ratios are evaluated[28] for the j-j coupling model, and they are for $P_{1/2} - P_{3/2}$ transitions:

$$\xi = -1/2 ,$$
$$\zeta = (2\lambda M)^{-1} ,$$
$$\eta = (9/64)\, R^2 |V| \Lambda ,$$

where $\Lambda = 2 \pm 1$. The nuclear radius for C^{12} is $R = 0.426 d\, A^{1/3} = 7.12\ (10^{-3})$. The potential energy V in the Coulomb field of the C^{12} nucleus is +6.15 for the positron and -6.15 for the electron.

As before, we define the recoil correction factor $C_o(Q)$ by

$$C_o(Q) = N(Q)/F_1(Q)$$

where $N(Q)$ is the total recoil spectrum, given by Eq. (50), and $F_1(Q)$ its dominant term. The opposite signs of $\underline{b}$ in the correction factors

$C_o(N^{12})$ and $C_o(B^{12})$ suggest that we consider their ratio, $C_o(N^{12})/C_o(B^{12})$, rather than each correction factor separately. In the ratio the effect of $\underline{b}$ is nearly doubled, and systematic errors of measurement made with the same instrument should tend to cancel.

The plot of the ratio, $C_o(N^{12})/C_o(B^{12})$, is shown in Figure 4. The values of $\underline{b}$ used are the estimates obtained from a least squares analysis of the Li^6 recoil data. The curves indicate that the momentum region of highest recoil probability corresponds to the steepest slope region of the correction factor ratio. This is fortunate, because it means that the experimental determination of $\underline{b}$ would be subject to smaller statistical uncertainties in that the less probable low momentum region of the recoil spectrum is much less sensitive to $\underline{b}$.

The measurement of the C^{12} recoil spectra from N^{12} and B^{12} beta decay is expected to be far more difficult than that of the Li^6 recoil spectrum. However, the ingenuity of experimentalists is never to be underestimated.

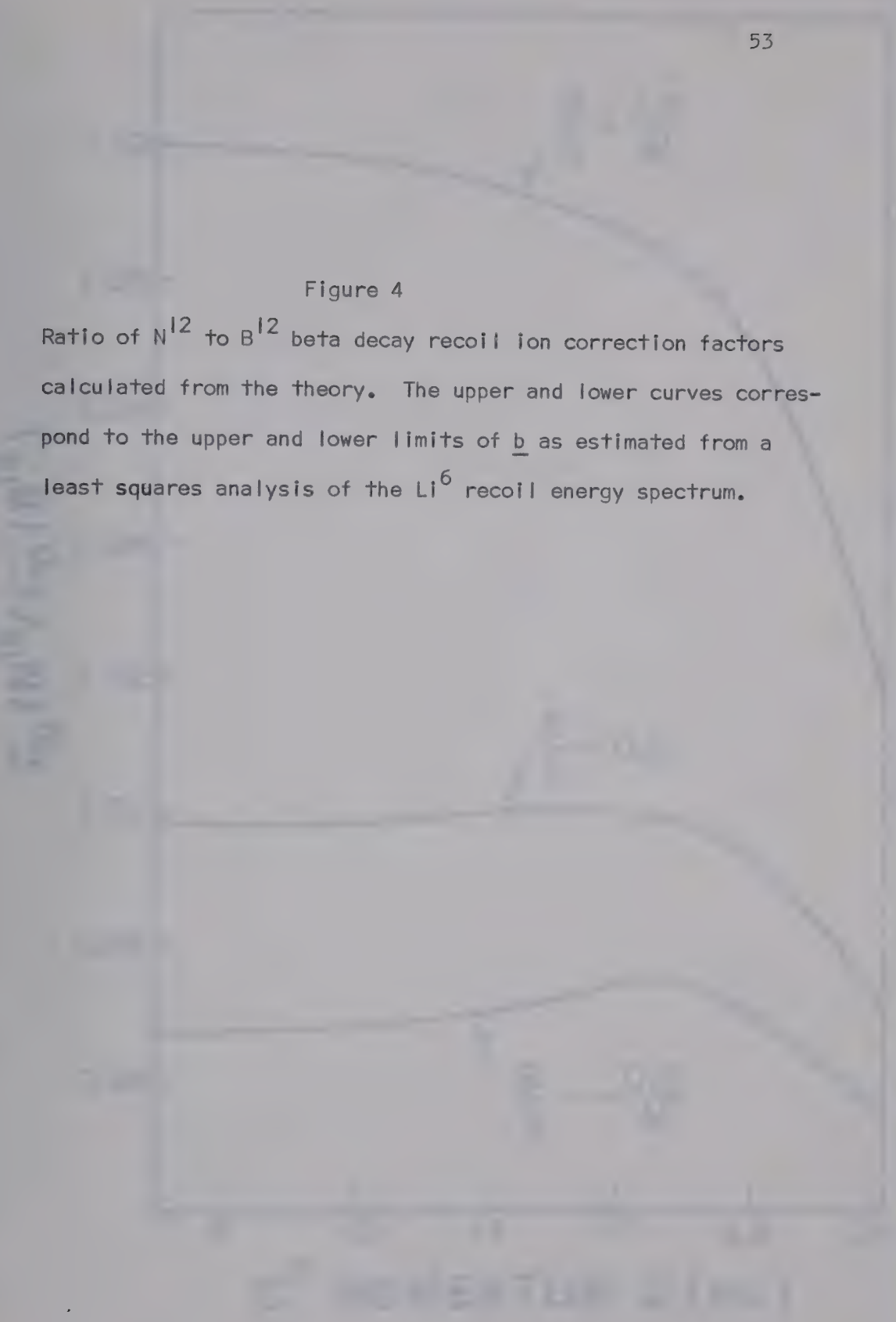
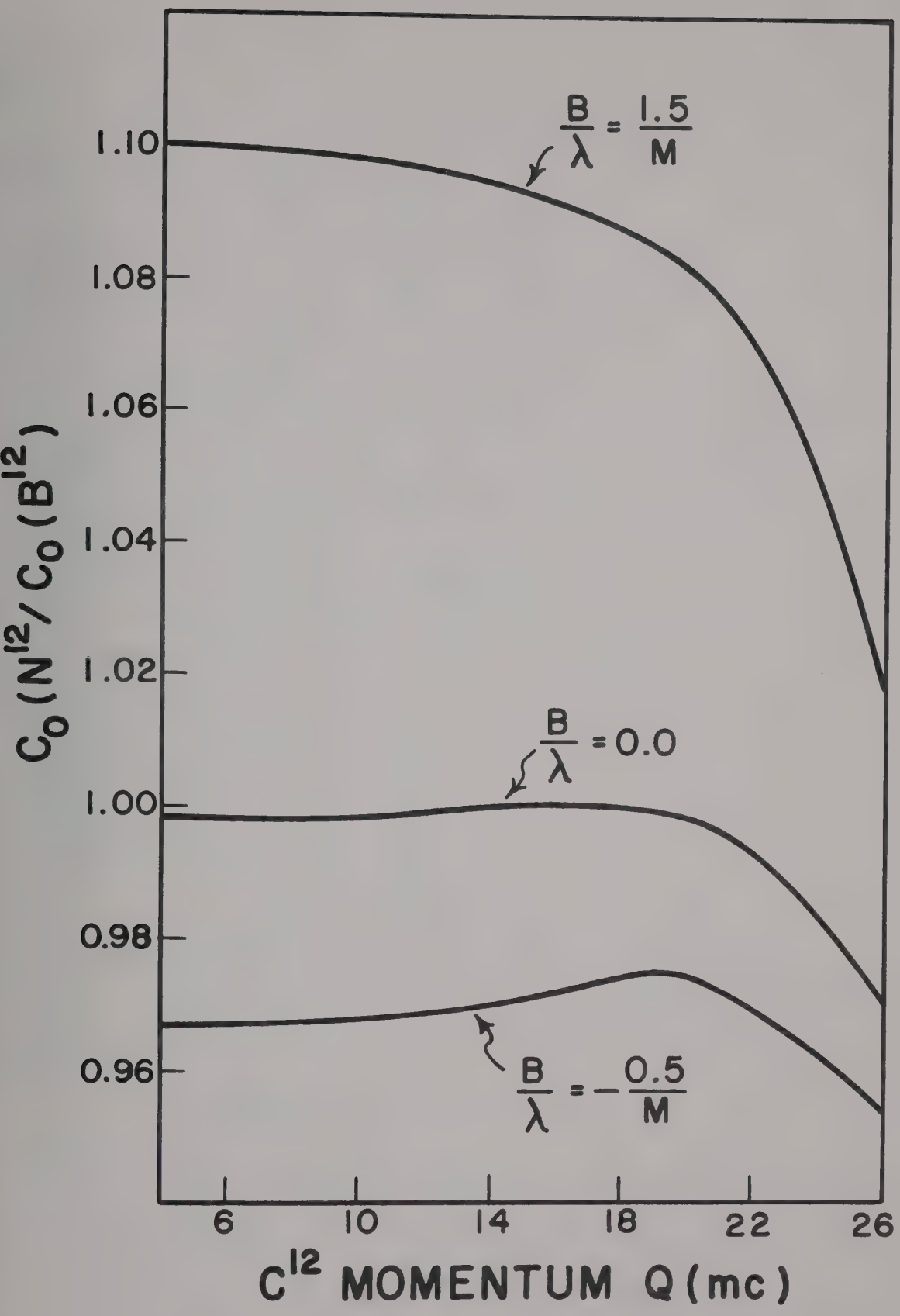


Figure 4

Ratio of N^{12} to B^{12} beta decay recoil ion correction factors calculated from the theory. The upper and lower curves correspond to the upper and lower limits of $\underline{b}$ as estimated from a least squares analysis of the Li^6 recoil energy spectrum.


$\frac{B}{\lambda} = \frac{1.5}{M}$
$\frac{B}{\lambda} = 0.0$
$\frac{B}{\lambda} = -\frac{0.5}{M}$
1.10
1.08
1.06
1.04
1.02
1.00
0.98
0.96
$C_0 (N^{12}/C_0 (B^{12}))$
6
10
14
18
22
26
C^{12} MOMENTUM Q (mc)

APPENDICES

Appendix A

ELIMINATION OF ODD NUCLEON OPERATORS

We demonstrate here explicitly the elimination of the odd nucleon operator γ_5, which, in the particular example of our consideration, occurs in combination with β and L_4 as $\beta\gamma_5 L_4$, where L_4 is the fourth component of the lepton field L_μ.

The Dirac equation for a nucleon is

$$H\psi = c\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta Mc^2 = i\hbar\frac{\partial\psi}{\partial t} \qquad (A.1)$$

where $\underset{\sim}{\alpha}$ and β are defined by

$$\underset{\sim}{\alpha} = \begin{bmatrix} 0 & \underset{\sim}{\sigma} \\ \underset{\sim}{\sigma} & 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \qquad (A.2)$$

Here, the components of $\underset{\sim}{\sigma}$ are the usual Pauli spin matrices.

The covariant form of Eq. (A.1), in the standard representation, is

$$\gamma_\mu\frac{\partial\psi}{\partial x_\mu} + M\psi = 0 , \qquad (A.3)$$

where

$$\underset{\sim}{\gamma} = -i\beta\underset{\sim}{\alpha} , \quad \gamma_4 = \beta , \quad x_4 = it . \qquad (A.4)$$

If we define γ_5 by

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} , \qquad (A.5)$$

Appendix A

Definition of the Model Equations

then $\underset{\sim}{\sigma} = -\underset{\sim}{\alpha}\gamma_5$. (A.6)

If we designate the Hermitian conjugate wave function by ψ^+ , and define the Dirac adjoint wave function $\bar{\psi}$ by

$$\bar{\psi} = \psi^+\beta \quad , \tag{A.7}$$

then the adjoint equation to Eq. (A.3) is

$$\frac{\partial\bar{\psi}}{\partial x_\mu}\gamma_\mu - M\bar{\psi} = 0 \, . \tag{A.8}$$

We now choose units such that $c = \hbar = 1$. In these units the Dirac free particle equation is simply

$$(\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta M)\,\psi = i\frac{\partial\psi}{\partial t} \quad , \tag{A.9}$$

and the hermitian conjugate is

$$\psi^+(\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta M) = -i\frac{\partial\psi^+}{\partial t} \, . \tag{A.10}$$

By multiplying both sides of the Eqs. (A.9) and (A.10) by equal quantities, we obtain the following equalities:

$$\bar{\psi}\beta(\beta\gamma_5 L_4)(\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta M)\psi = i\,\bar{\psi}\beta(\beta\gamma_5 L_4)\frac{\partial\psi}{\partial t} \tag{A.11}$$

$$\bar{\psi}\beta(\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta M)(\beta\gamma_5 L_4)\psi = -i\,\frac{\partial(\bar{\psi}\beta)}{\partial t}(\beta\gamma_5 L_4)\psi \tag{A.12}$$

Since α_x , α_y , α_z , β , and γ_5 are anti-commuting matrices whose squares are unity, Eqs. (A.11) and (A.12) can be reduced to :

$$\bar{\psi}[\,\gamma_5 L_4\underset{\sim}{\alpha}\cdot\underset{\sim}{p} - M\beta\gamma_5 L_4\,]\psi = i\,\bar{\psi}\gamma_5 L_4\frac{\partial\psi}{\partial t} \tag{A.13}$$

$$\bar{\psi}[\,-\underset{\sim}{\alpha}\cdot\underset{\sim}{p}\,\gamma_5 L_4 + M\beta\gamma_5 L_4\,]\psi = -i\,(\frac{\partial\bar{\psi}}{\partial t})\,\gamma_5 L_4\psi \, . \tag{A.14}$$

Now, by subtracting Eq. (A.14) from Eq. (A.13), we get

$$\bar{\psi}[\,\gamma_5 L_4\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \underset{\sim}{\alpha}\cdot\underset{\sim}{p}\,\gamma_5 L_4 - 2M\beta\gamma_5 L_4\,]\psi = i\,[\frac{\partial}{\partial t}(\bar{\psi}\gamma_5\psi)]L_4 \, . \tag{A.15}$$

$\frac{\partial}{\partial t}(\bar{\psi}\gamma_5\psi)$ is negligible[22]. $\underset{\sim}{p}$ is a gradient operator that operates on all functions of x to its right. We also recall that $\underset{\sim}{\sigma} = -\underset{\sim}{\alpha}\gamma_5$. Thus we obtain

$$\bar{\psi}[\beta\gamma_5 L_4]\psi = -(1/M)\bar{\psi}[\,L_4\underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,]\psi + i(1/2M)\bar{\psi}[\underset{\sim}{\sigma}\cdot\underset{\sim}{\nabla}L_4]\psi .$$

Appendix B

METHOD OF LEAST SQUARES

The method of least squares is a very powerful method for obtaining the most reliable possible information from a set of experimental observations. It seems natural, therefore, that we choose this particular method for the estimation of the IT coupling constant, b, from the Li^6 recoil data.

The principle of least squares can be stated as follows: The most probable value of a quantity is obtained from a set of measurements by choosing the value which minimizes the sum of the squares of the deviations of these measurements. Since our primary interest is the determination of b, it is convenient if we single out the IT coupling term and write the total theoretical spectrum as

$$N(E) = F(E) + b\,G(E)$$

where G(E) is the IT coupling term and F(E) is the sum of the dominant and all other small terms. In order to relate the theory to experimental observations we normalize the theoretical spectrum such that

$$C\,N(E) = U(E) \pm \sigma$$

where C is the normalization constant and U(E) is the experimental spectrum. It is also convenient if we define $Cb = b'$.

We now construct a function of two variables, $\chi(C,b')$, such

that

$$\chi^2(C,b') = \sum_{k=1}^{16} [\, CF_k(E) + b'G_k(E) - U_k(E) \,]^2 / \sigma_k^2 \quad .$$

Then we minimize $\chi^2(C,b')$ with respect to the two independent variables, which yields:

$$\sum_{k=1}^{16} [\, CF_k + b'G_k - U_k \,] F_k / \sigma_k^2 = 0 , \qquad \text{(i)}$$

$$\sum_{k=1}^{16} [\, CF_k + b'G_k - U_k \,] G_k / \sigma_k^2 = 0 , \qquad \text{(ii)}$$

From Eqs. (i) and (ii) we can solve for C and b'. The quantity of interest, $\underline{b}$, is then obtained from the ratio of b' to C, which, in turn, can be computed from:

$$b = \frac{b'}{C} = \frac{[\sum_{k=1}^{16} F_k^2/\sigma_k^2][\sum_{k=1}^{16} G_k U_k/\sigma_k^2] - [\sum_{k=1}^{16} F_k G_k/\sigma_k^2][\sum_{k=1}^{16} F_k U_k/\sigma_k^2]}{[\sum_{k=1}^{16} G_k^2/\sigma_k^2][\sum_{k=1}^{16} F_k U_k/\sigma_k^2] - [\sum_{k=1}^{16} F_k G_k/\sigma_k^2][\sum_{k=1}^{16} G_k U_k/\sigma_k^2]} \quad .$$

(i)

(ii)

(ii) ... and b'. The quantity of ...

... is then obtained from the ratio of b' to ..., which, in turn, ...

$$\frac{\qquad\qquad}{\qquad\qquad} = \frac{\qquad}{\qquad}$$

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BIOGRAPHY

Sung Kyu Kim was born on January 12, 1939, in Korea. He came to the United States of America in December 1954. After two years in high school, he entered Davidson College in 1956, and received the B. S. degree in 1960. His undergraduate honors include election to Phi Beta Kappa and a summer research assistantship at Lamont Geological Observatory, Columbia University.

During the years 1960-1962 he was a Gilman fellow and a junior instructor in physics at The Johns Hopkins University. He entered the Graduate School of Arts and Sciences of Duke University in September 1962. At Duke University he held a graduate teaching assistantship from February 1963 to May 1964, receiving the A. M. degree in Physics in June 1964. Since then, he has been a research assistant in theoretical physics. He is at present (July 1965) a candidate for the Ph. D. degree at Duke University.

He has been appointed Assistant Professor of Physics at Macalester College effective September 1965.